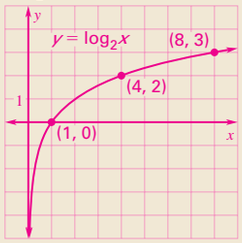
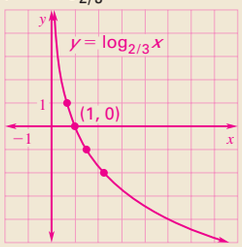
Extra Example 7

Graph the function.

a. $\log_2 x$



b. $y = \log_{2/3} x$



Key Question to Ask for Example 7

- What two points are on the graph of any logarithmic function of the form $y = \log_b x$? **(1, 0) and (*b*, 1)**

Animated Algebra classzone.com

An **Animated Algebra** activity is available on-line for **Example 7**. This activity is also available on the **Power Presentations CD-ROM**.

Avoiding Common Errors

Some students may have trouble making a table to values to use for graphing a logarithmic function. Because students at this point are more familiar and comfortable with exponential functions than logarithmic ones, show them how they can use what they already know. Suggest making a table of values by choosing the y-values first, using consecutive integers, and then finding the corresponding x-values to form the ordered pairs. A similar approach is to construct a table for the corresponding exponential function and then to reverse x and y in each pair to find ordered pairs for the logarithmic function.

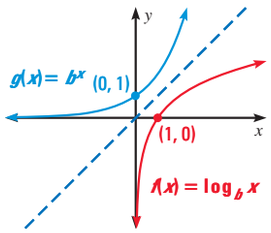
GRAPHING LOGARITHMIC FUNCTIONS You can use the inverse relationship between exponential and logarithmic functions to graph logarithmic functions.

KEY CONCEPT *For Your Notebook*

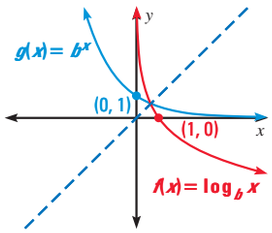
Parent Graphs for Logarithmic Functions

The graph of $f(x) = \log_b x$ is shown below for $b > 1$ and for $0 < b < 1$. Because $f(x) = \log_b x$ and $g(x) = b^x$ are inverse functions, the graph of $f(x) = \log_b x$ is the reflection of the graph of $g(x) = b^x$ in the line $y = x$.

Graph of $f(x) = \log_b x$ for $b > 1$



Graph of $f(x) = \log_b x$ for $0 < b < 1$



Note that the y-axis is a vertical asymptote of the graph of $f(x) = \log_b x$. The domain of $f(x) = \log_b x$ is $x > 0$, and the range is all real numbers.

EXAMPLE 7 Graph logarithmic functions

Graph the function.

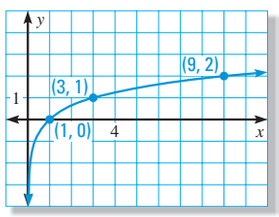
a. $y = \log_3 x$

b. $y = \log_{1/2} x$

Solution

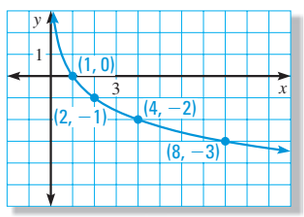
a. Plot several convenient points, such as (1, 0), (3, 1), and (9, 2). The y-axis is a vertical asymptote.

From *left* to *right*, draw a curve that starts just to the right of the y-axis and moves up through the plotted points, as shown below.



b. Plot several convenient points, such as (1, 0), (2, −1), (4, −2), and (8, −3). The y-axis is a vertical asymptote.

From *left* to *right*, draw a curve that starts just to the right of the y-axis and moves down through the plotted points, as shown below.



Animated Algebra at classzone.com

Differentiated Instruction

Visual Learners Reinforce the inverse relationship between $y = \log_b x$ and $y = b^x$ by having students graph the reflection in the line $y = x$ of each of the graphs in **Example 7**. Have students write and graph each inverse function and compare each graph to the graph of the reflection.

See also the *Algebra 2 Toolkit* for more strategies.

TRANSLATIONS You can graph a logarithmic function of the form $y = \log_b (x - h) + k$ by translating the graph of the parent function $y = \log_b x$.

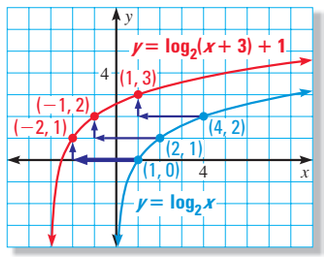
EXAMPLE 8 Translate a logarithmic graph

Graph $y = \log_2 (x + 3) + 1$. State the domain and range.

Solution

STEP 1 **Sketch** the graph of the parent function $y = \log_2 x$, which passes through (1, 0), (2, 1), and (4, 2).

STEP 2 **Translate** the parent graph left 3 units and up 1 unit. The translated graph passes through (−2, 1), (−1, 2), and (1, 3). The graph's asymptote is $x = -3$. The domain is $x > -3$, and the range is all real numbers.



GUIDED PRACTICE for Examples 7 and 8

Graph the function. State the domain and range. 16–18. See margin for art.

16. $y = \log_5 x$
domain: $x > 0$,
range: all real numbers

17. $y = \log_{1/3} (x - 3)$
domain: $x > 3$,
range: all real numbers

18. $f(x) = \log_4 (x + 1) - 2$
domain: $x > -1$,
range: all real numbers

7.4 EXERCISES

HOMEWORK KEY
○ = WORKED-OUT SOLUTIONS on p. WS1 for Exs. 13, 33, and 61
★ = STANDARDIZED TEST PRACTICE Exs. 2, 36, 61, and 62

SKILL PRACTICE

A

1. **VOCABULARY** Copy and complete: A logarithm with base 10 is called a(n) ? logarithm. common

2. ★ **WRITING** *Describe* the relationship between $y = 5^x$ and $y = \log_5 x$. The functions are inverses of each other.

EXAMPLE 1 on p. 499 for Exs. 3–7

EXPONENTIAL FORM **Rewrite the equation in exponential form.**

3. $\log_4 16 = 2$ $4^2 = 16$

4. $\log_7 343 = 3$ $7^3 = 343$

5. $\log_6 \frac{1}{36} = -2$ $6^{-2} = \frac{1}{36}$

6. $\log_{64} 1 = 0$ $64^0 = 1$

7. **ERROR ANALYSIS** *Describe* and correct the error in rewriting the equation $2^{-3} = \frac{1}{8}$ in logarithmic form. See margin.

$\log_2 -3 = \frac{1}{8}$ ✗

EXAMPLE 2 on p. 500 for Exs. 8–19

EVALUATING LOGARITHMS **Evaluate the logarithm without using a calculator.**

8. $\log_{15} 15$ 1

9. $\log_7 49$ 2

10. $\log_6 216$ 3

11. $\log_2 64$ 6

12. $\log_9 1$ 0

13. $\log_{1/2} 8$ −3

14. $\log_3 \frac{1}{27}$ −3

15. $\log_{16} \frac{1}{4}$ $-\frac{1}{2}$

16. $\log_{1/4} 16$ −2

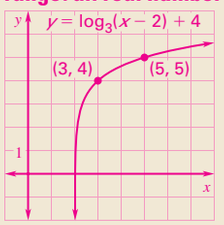
17. $\log_8 512$ 3

18. $\log_5 625$ 4

19. $\log_{11} 121$ 2

Extra Example 8

Graph $y = \log_3 (x - 2) + 4$. State the domain and range. **domain: $x > 2$; range: all real numbers**



Key Question to Ask for Example 8

- Do translations of a logarithmic graph affect the domain and/or the range? Explain. **A horizontal translation affects the domain, but not the range. A vertical translation does not affect the domain or range, since the range is always all real numbers.**

Closing the Lesson

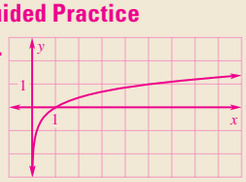
Have students summarize the major points of the lesson and answer the Essential Question: What is the relationship between exponential and logarithmic functions?

- **The equations $\log_b y = x$ (logarithmic form) and $b^x = y$ (exponential form) are equivalent.**
- **The graph of a logarithmic function rises from left to right if $b > 0$ and falls from left to right if $0 < b < 1$. The graph has a vertical asymptote.**

Exponential and logarithmic functions with the same base are inverses.

7. ***Sample answer:*** **The −3 and $\frac{1}{8}$ are switched around; $\log_2 \frac{1}{8} = -3$.**

Guided Practice

16.


17.


18.